Creativity in music and math
Pierre Boulez and Alain Connes

INTRODUCTION : Good evening everyone, welcome to the heart of IRCAM in the projection space, for this original meeting between a mathematician, Alain Connes, and a composer, Pierre Boulez. So, this meeting belongs to the Agora festival, which questions the relationship between invention and constraint, finally between intuition and logic. And it seemed very important to us to place this nodal meeting point this evening, this attempt to meet between two worlds which coexist and which, maybe, have things to say to each other.

So I just wanted to point out that obviously there will be deduction in artistic operation as well as intuition in mathematical operation. And it's an unfathomable relationship and quite complex. Gérard Assayag, director of the Joint Research Unit CNRS-Ircam, will lead, if necessary, this debate, in any case, will serve as a catalyst. And I also wanted to say that this debate is part of the Mathematics and Music Conference, an international conference taking place at this time at Agora.

Maybe this conference will decree the irreducibility between artistic invention and mathematical invention. But irreducible is a term which was questioned by mathematicians. So we stay in the mathematical field. By way of launching, I only wanted to make one quote, like we often do in France to start or to finish, a quote of the most intuitive, and maybe of the most deductive of all minds, Leibniz, who said and who certainly spoke to composers as much as to scientists :

> *"The perfect world is the simplest world in hypotheses and the richer in phenomena.".*

I give the floor to Gérard Assayag.

This conference was held at IRCAM on June 15, 2011. It can be viewed at the address : https://medias.ircam.fr/x70ce3e$_{p}ierre - boulez - et - alain - connes - la - creativ$

Gérard Assayag : Thank you Franck. We will start with a short presentation by Pierre Boulez and then engage in a dialogue in partly, but in partly only, improvised.

Pierre Boulez : Okay, so a little text at the beginning, to launch a little debate, because it is not at all a definitive and dogmatic text. It is a text, on the contrary, rather skeptical, I would say. If, to account of a work, we talk about mathematical music, it is not an alloy very cordial. These two words, so close to each other, indicate a work barbative, dry, inexpressive, boring.

It does not come from the heart, does not return to the heart, to quote, once in addition, this great model, but comes out of the brain and doesn't even go to another brain. So it's already a kind of rehabilitation of thinking, of musical reflection than one way of directly bringing the two words mathematic, music, and to add the third word contact, discrete word, unpretentious, but a sign of a will that could not be more determined. Good. Obviously, this is not the first time that this rapprochement has been attempted.

From the quadrivium in the Middle Ages, to the work of Rameau and d'Alembert and even the mystical constructions of Scriabin. We even have much written. And yet, there is still some sort of border, said, between musical creativity and the structure of the language explained or the less scientifically approached. When a musician, a composer, close to the computer tool, who wishes to use electronic equipment, many misunderstandings can arise, which are difficult to overcome. Desiring before all the tools that allow him to work step by step, he attaches himself to an immediate return. He expects to be made proposals, let him to be given examples. From there, he can imitate these examples or try to transgress them by modifying the parameters which one proposes to him. But he may as well not go further and abandon this tool that he has just touched. The second pitfall is to transcribe too literally rather, diagrams supplied to it by the mathematical tool or arithmetic.

That which in a case makes sense is no longer relevant and does not make sense in the literal transcription. As much the first approach that I point out is based on immediate perception and does not care to codify for a launch pledge, as much the second approach worries very little, if at all, about the perception, and relies much more on the notion of schema that can be applied regardless of any parameter. Taking into account only the perception, we cannot organize a language, the objects that we found are not strong enough for that. If you don't take perception into account, language can only be constituted in a properly hazardous manner, the parameters not having the same value in the template and in the transcript.

This is where the aesthetic criterion appears to choice or reject the so proposed solutions ? Facing the picture was total possibilities. Intuition becomes like an indispensable short circuit. This is how among all possible universes of intervals, durations, dynamics, etc., intuition is going to choose the one that will serve the composer when the solution will acquire all its necessity. The more we will be able to master this universe of possibilities, the more the intuition will have been used as an absolute criterion in this instant of the choice more or less approached a certain truth that we need at a given time.

Besides, whether we think music with or without an interpreter, music combined between electronics and instrument or purely electronic music, it remains to find the gesture and the form. We are no longer dealing with objects, but with textures which, by continuously changing or breaking, will occupy a space-time. What mathematical model will give us the possibility of finding this gesture which will justify all the other categories ?

From this point of view, I found the quote from Malarmy placed at the head of this symposium : "A dice will never abolish the hazard.". To sum up my attitude as a composer, I would say that I did not wait the whole from a systematic organization of a few parameters that these are. I suppose that the invention, if it is carried out, can only be done if it admits the accident, the unexpected that questions to what we thought establish.

As far as I can tell, scientific intuition goes through the same phases. And on this uncertain ground, it is able to confront with musical intuition. It is a very fragile profession of faith, of course, that I propose, but I owe, I believe, more to this fragility than to the security of dogmas. I believe it to be full of promise.

Gérard Assayag : Your conclusion illustrates a tension which, I think, crosses your work, which is the tension between system and freedom. And in a recent interview with the magazine Musik Blätter, returning to *The hammer without master*, you clearly indicated how this work had marked its time by a combination of very finished constructivism, even a little rigid, stemming from the school of Vienna, but with an ornamental freedom and a certain freshness that we could call the French spirit let's say. So the idea was to work with constructivism, but so as to be free there. It's one thing which is not obvious and I say to myself that it may be a problem that also the mathematician meets. What do you think, Alain Connes ?

Alain Connes : Let's say that I've thought about these two things a bit. There are aspects of which we speak relatively little in mathematics, which are precisely creativity and the role of aesthetics. And I think I'll deliver some thoughts I had on that, but just like a starting point, I think it will match what you have told. So, in fact, a priori, when we talk about creativity in mathematics, the mathematician is a little skeptical because most of the task of the mathematician is problem solving. And it's basically a task of discovery. That is to say, the mathematician is looking for truths, that preexist his presence, before he begins to search. And what is quite extraordinary, precisely, you were talking about this relationship with mathematics, which is quite extraordinary, is to see that mathematics evolution that took place in the XX$^{\text{th}}$ century, in fact, already allows the close relationship between music and mathematics. Why ? Because, in fact, the role of mathematics which, at the beginning, was a role that one could roughly summed up as part of physics, has become, over time, in a thematic called modern mathematics of XX$^{\text{th}}$ century in fact, it has become a kind of substitute for philosophy at the level of concept creation. And what's quite remarkable, in fact, is that so far, this transition can almost be traced back to Galois. And which is quite remarkable, is that a bit like in music, it has generated at the start considerable resistance which continues to manifest itself sporadically. But I'll quote you... Does it bother if I do a quote in English because it's a text that is in English at the beginning. But it is a very recent text by a well-known mathematician who shovel Vladimir Arnold, and who talks about mathematics, and who talks about the teaching of mathematics, and who speaks about modern mathematics. Don't worry, I'm French, so I will defend the French point of view after. But I still have to expose this point of view. So he says :

"*Mathematics is a part of physics, physics is an experimental*

> *science, a part of natural science, mathematics is the part of physics where experiments are cheap.* (laughs). *In the middle of the twentieth century, it was attempted to divide physics and mathematics. The consequences turned out to be catastrophic. Whole generations of mathematicians grew up without knowing half of their science, and of course in total ignorance of any other science. They first began teaching their ugly scholastic pseudo-mathematics to their students.*"

It continues, it continues, and his text is very funny, it is full of spikes, etc. And then, he says :

> "*The ugly building built by under-educated mathematicians who were exhausted by their inferiority complex and who were unable to make themselves familiar with physics, reminds one's...*"

Well, then after, he speaks of an axiomatic of odd numbers, etc. and then he says so, finally, that he interviewed French math students for example mathematics students, he asked them "2 + 3 ?". And a french primary school pupil replies "3 + 2 because addition is commutative". And then he explains :

> "*Judging by my teaching experience in France, the university students'idea of mathematics, I feel sorry for them because they are very intelligent but deformed kids, is as poor as that of this pupil.".*"

The student who answered "2 + 3 = 3 + 2.". And then he give examples.

But in fact when we deepen this text by Arnold a little, we realize if he wants, what he criticizes is mathematics. What they criticize is if you want all the examples he takes where he says the modern mathematicians don't know how to do that, etc., it's mathematics XIX$^{\text{th}}$ century and the mathematics, he gives examples of curves plot in the plane or things like that, it's math that now are completely digested and the computer does a lot better than a mathematician, he does it in a quarter of a second. And what he did not digested, what he doesn't explain is the wonderful phenomenon which occurred in the mathematics of the XX$^{\text{th}}$ century and which precisely, allow

... I'll read you a little text from Grothendieck. And what Grothendieck says is :

> "*The progressive clarification, precisely, of the notions of definitions, statements, demonstrations, mathematical theories including everything we could, if we only did mathematics as being a part of physics, we could ignore them completely. And to say that it is fantasies of axiomaticians, was in this respect very beneficial and made us become aware of the power of childlike simplicity however. That is to say that mathematical concepts, in fact, should not to fear. In general, they have a childish version and this childish version is much closer to their reality than the extremely versions elaborate," therefore of a childish simplicity, however, which we have to formulate with perfect precision the very people who might seem indivisible by virtue of a sufficiently rigorous use of language more or less current pledge. If there is one thing that fascinated me about mathematics since my childhood, it is precisely this power to define in words and to express perfectly the essence of such mathematical things which, at first glance, appear in such an elusive form or so mysterious that they seem beyond words.*"

And that, if you will, is an extremely important thing because most people, when you talk to them about math, they think about arithmetic, they think of numbers.

Okay, they may be thinking about geometry, but they don't realize that modern mathematics, that is to say the mathematics of the XX$^{\text{th}}$ century, they have just succeeded in perfecting the current language by concepts which are extremely precise, but which have potential of applications that goes far beyond physics.

So, when you think about music and if you want, for well situating things in relation to mathematics, I'm going to read you a little text that I wrote a long time ago and where I was talking about the link in between and I said,

> *"It is crucial to me for a child to be exposed to music very early. I think exposing a child to music at the age of 5 or 6 makes it possible to balance a little the preponderance his sense of sight intel-*

lect and this incredible, purely visual wealth, that a child acquires very early and which, therefore in fact is related to geometry.".

It is linked to geometry as long as it fits into space through a mental image. If you want, there is the same phenomenon in mathematics than in relation to a musician. When a non mathematician sees a mathematician working in the metro. What does he see ? He sees a page full of formulas. They have no meaning. When a non-musician sees a musician working in the metro and reading a partition, it is exactly the same, it feels like ... it's the same ! Now, there is an essential part of the work of mathematicians which is precisely to create mental images. But when I talk about mental images, it has to do with geometry. We see a geometric figure, we see, it fits into space. But what's really amazing is that so far, in the functioning of the mathematician, there is not only the geometric image, there is algebra. And there is nothing visual about algebra, but on the other hand, algebra has a temporality, that is to say that the algebra fits into the time.

When you do a calculation, when you expose a demonstration, that takes place over time. It's just like the musician who, after having understood a musical work, having it completely zipped in his mind, something that has nothing to do with it, spreads it out. For the mathematician, it's the same. When he does an algebraic calculation, it takes place over time, but it's something which is very close to language, which has this diabolical precision of language and in a way, if you will, there is a pretty incredible collusion between algebraic computation, that part of mathematics that has to do with the language, which takes place over time and certain musical works.

And that, I can't help thinking about it. In other words, for me, there are some relatively short musical works that say something. And I even had this impression, you will laugh but I even had that impression when we saw these rooms that go repeatedly listen to Beethoven's sonatas of years and years. It reminded me, if you will, people who are there and who are trying to understand. And we repeat the same to them. They know there is something, and this thing is not transferable by another way than through music. We can't transform into something else than what is transmitted, but it transmits something.

So, we cannot say that it is not a language. Likewise, we can't say that maths don't have a language aspect. They have a language aspect which is extremely important.

But the gist of what I said, if you like, is that this aspect launched pledge of mathematics has become much more flourishing. He became much more expressive. It has become much broader than, precisely, the mathematics of the XIX$^{\text{th}}$ century. And when we stay with mathematics of the XIX$^{\text{th}}$ century, of course, you can say "Oh, these mathematics have a connection with music because there is arithmetic, there is log 3 on log 2, which is the well-tempered keyboard, etc."

But it will not go beyond. In fact, mathematical language, so far has crossed many other frontiers and in a way, now, we can hope that, precisely, there is a possibility of reconciliation which is much bigger because of that. People still must accept mathmodern materials. And people still must have absorbed all this elaboration, which is not at all obvious.

Gérard Assayag : So this algebra / geometry duality is one of your workhorses. It's extremely interesting because it brings to the heart of the math-music problem, because it's a duality that we constantly meet in musical research, namely metaphorically, we discussed it, either technically, and I could possibly give examples. So especially in musical analysis, that is to say that when we look at a score, there is an expression which I heard and which I like well, it is the partition seen plane, when we try to understand this mechanism, that is to say it keep as a whole but we have the right to do what we want. We can jump from one point to another and relate a point to another freely and it's an obviously geometric vision.

Alain Connes : Of course.

Gérard Assayag : Or there is another way of approaching it, which is from the point of view of these generating mechanisms. And here we have a point much more local, because we look at the mechanics. Is that, it's something you feel, this tension when you, when you watch music, not when you create it, but when you watch an existing music ?

Pierre Boulez : When I watch music, I start first by trying to understand the form, because it is what directs you, simin the evaluation. We did one experiment here out of three levels of understanding of music. We first gave, say, a Mozart's sonata or a movement, or a half-movement (the first half of the movement), and we asked someone who is absolutely not a musician, someone who has no musical culture, we asked him what did he think ? He gave a very vague description of it and not at all relevant if I can say. Then the medium level. He had listened several times sonatas by Mozart, so he could find a form, at least a contrast between themes. It's already a much more precise and the cultured, then, described exactly what happened. Then we spent a Schophausen's piano work, a fragment of course. Well, the three responses were very similar because everyone created their own theater of the shape and they would point to the passages that had particularly struck them, that is, there was no conception of form.

But there was a conception of events and events that were still not linked by a form but separate events, which had struck, either because they were very strong, or because they were played by a specialized instrument, etc., etc. So you can see that it's very difficult to approach even a form, because a form is really, disounds, what... how the person looks at it. And there, when one is a musician, obviously, we are trying to have a view, let's say more objective, and not only subjective.

This is how I see music. So when we see the detail, we indeed see how the discourse is constructed and if it is constructed more horizontally than vertically or more vertically than horizontally, or if it is built by breakage, or if it is built by continuity, etc., etc. There is a lot of ways of looking at the very perception of music and I'm convinced that there are many people who also make a kind of... who... since they cannot understand the musical form, who make themselves a certain narration, especially when they have listened to a work several times, they make a personal narration and it is this narration that they follow. That's why people, the general public, if they don't make an effort, settles so well in a work because they always listen to it by the same way and therefore that they has before theim the same images, the same stereotypes, the same shots, I would say, rather than the same pictures. And that's how they absorb music, they don't absorb it by a kind of description of continuity, they accepts it as a whole, followed by a whole, followed by a whole.

Gérard Assayag : But the expert-analyst, the composer who watches another composer, doesn't he have this freedom when he is looking at a score, to finally see it as a space where he can walk around at will which is not realistic insofar as the score was not generated from that way, by acting simultaneously on all parties ?

Pierre Boulez : Yes, certainly, when you analyze ... Me, what interested me in analysis, it's even false analysis, but which generates something.

I remember once when Stockhausen showed me an analysis of the Webern quartet, but he looked at the density of meetings. What has nothing to do with Webern, which was just a four-voice counterpoint, and therefore a four-way counterpoint, especially if it's a cannon, things are offset from each other. So if there is an incorrect individual phrasing, things are obviously not always of constant intensity. But for him, what interested him at the time was the phenomenon of intensity.

How can a four-voice cannon give intensities of this order, statistically speaking. I find it more interesting than analyzing even simply how the composer designed it. What is interesting in an analysis, it's not when you want to redo what the composer has done, it is to see by what process he arrived at such a result. And so, even if the analysis is false, is completely false, the analysis is much more interesting because it is productive.

Alain Connes : Okay, there is still a rather frank difference, precisely, there, we are talking about Works. So we see... So if we look at a particular aspect of mathematics, which is a demonstration, we can say the next thing which is a little bit similar, it is that if you want there are two ways to look at a demonstration. There is a check line by line. And that, I think, is a bit like someone playing a song of music which he has not yet digested and who is obliged to have the score before the eyes.

So we can do that. We can check a demonstration line by line. But there is a second step which is extremely important. Because in fact, a mathematician knows he only understands a demonstration when he is able in his brain to zip it in half a second. That is to say that he will not have the successive ingredients of the demonstration, but he will have immediately the entire demonstration.

Pierre Boulez : Can I open a parenthesis ?

Alain Connes : Of course.

Pierre Boulez : In music, that very much depends on a very different point of view , according to the fact you are a performer, or if you are a composer. If you are a composer, you have plenty of time to navigate and you move from one point to another and you try to consolidate your analysis by comparison from one point to another, what are the differences, what are the similarities, etc. If you're a performer, this way, let's say, of amassing the knowledge is a consequence ... is a kind of unconscious thing.

When you are at point D, for example, you know that you have already played point A, and its successions, and you know you're going to meet the point N and its successions. But you don't know exactly. But you know, the closer it gets, the more you are aware of what will follow. And the more it goes away, the more you are aware that it goes away and therefore that the form has reached a point of the present, that is to say that we constantly have these three dimensions in the head, present, of course, where you are and the past who brought you there, the future which will lead you to ...

Alain Connes : Of course, of course. But what I mean is that precisely, this kind of linearity of the work, there is something that is extremely striking for the mathematician, that is to say that if a mathematician tries to understand a demonstration, there is this process which is to try to read it linearly. There is another process which is much more efficient, which is to look at the theorem statement and start by looking for a demonstration yourself.

And when we've done that, what happens is that reading the demonstration, at that time, we will say : "But that's nothing. That's nothing". And we are going to say : "There, there is something going on". And it's only like that, it's only from this mechanism that we really understand what is going on... So that, I don't know if there is something analogous to that in a musical work. That is to say, does a musical work answer a question, etc. And we can say when the work takes place "Ah !". Well, I sometimes had that impression at the end of certain pieces where there was a kind of moment when there was a moexplanatory a posteriori or vice versa. I mean by the time we see that there is a theme that will then unfold, etc. But in mathematics, it is something extremely strong.

That is to say, there is a huge difference, precisely, between mathmatician who vaguely understands the statement and then begins to check the demonstration step by step, etc. And the mathematician who is going to have an act which is not at all passive, but will start to think for himself and after, only after, go and watch the demonstration.

GÉRARD ASSAYAG : Does that have anything to do with compression, this zipping you are talking about ?

ALAIN CONNES : Absolutely, of course, of course. That is to say, the mathematician works by levels of abstraction, by hierarchical levels of abstraction, that is to say that in fact, it means that he cannot progress, as the concepts are very complicated, and with this notions of zipping, he is able to make them occupy a space which is almost zero and afterwards, he will be able to manipulate them abstractly without knowing what the zipped contains, simply by having an intuitive idea of "what this motion signify. Of course for that, language is extremely important, that's why, well, there are very creative mathematicians like Grothendieck, etc. who gave 36 new names like *schema*. Schemas have a very precise mathematical sense, etc. And it's only with this zipping mechanism that we can progress through hierarchical levels of understanding.

PIERRE BOULEZ : For music, it is mainly memory that plays a role. I see, for example, it's very striking, when I was mainly in orchestral charge, I did introductory sessions, but explanations on musicians'Works and I always noticed that there was always a need for examples. By that, when you play the work, the example immediately comes to mind.

And there, the memory works so as to magnetize the perception in a direction or in another.

ALAIN CONNES : Okay, yes, so I think there is something which is very analogous in this case, because, well, there are some mathematicians like Grothendieck who work a little bit backwards, that is to say that they start from the general case and then they... but most of the mathematicians work differently, that is, if they are given a good example and have been explained something concretely on an example, a general phenomenon, precisely, they are perfectly capable to immediately generalize and to have the general case and I guess that in musique, finally, we can see in Beethoven's music or things like that, we can see that there is a generative system that allows from things relatively simple to generate quantities of things which are deduced from it and this, in mathematics, is a fairly general phenomenon. So there is this side of almost automatic generation that occurs and that plays a very important role, very, very important.

Gérard Assayag : So to come back to this duality concerning algebra versus geometry, you mention, so it's very important, that on the algebraic side, you put time, there is a begetting, and so a generation. There is a combinatorial of symbols. There are production rules. These are things that we use a lot in music. The musicians were interested a lot, for example, in formal grammars or production rules to find interesting sequences, or not elsewhere, of notes. But as soon as it produces sequences, we agree, but sequences are they sufficient to define time ? It's a question that I'm going to ask both to the mathematician and to the musician.

Alain Connes : Of course, I will answer because I mean : my first mathematical work consisted exactly in that, that is to say if you want, and what is quite incredible, is that, precisely, we realize that this called non-commutativity, what does that mean ? It means that when you write a word, it's not all of the letters of the word that matters, but it's also the order in which it is written. Okay, well, we can give 36 examples. And what is absolutely unbelievable, what is absolutely incredible, is that precisely, you realize when you do math, you realize that when you look at non-commutative geometry, that is to say the algebra precisely, in which one does not dare to say that $abab$ is equal to a^2b^2, well, time is spawned in a natural way. This is much stronger than saying that algebra takes place over time.

In fact, and that comes from the quantum, that is to say the quantum taught us that precisely, when we were doing mechanical calculations, in a quantum paradigm, we couldn't, that's what Heisenberg found, we could not swap quantities like position and time, etc. We could no longer calculate too simply when we are interested in microscopic systems, which is absolutely amazing. And the philosophical potential has not been sufficiently exploited at all, therefore. The fact is that when we take an algebra of a certain quality, which we call an operator algebra, which is non-commutative, well, it generates its own time. It has a group of automorphisms which is parameterized by a parameter t but that is really the time in the physical examples which turns over time. So this is amazing and it comes exactly because you cannot swap a and b. So when you write a word, the order of the letters is important, while when Descartes, etc., when people of that time were doing calculations, they were doing calculations commutatively, that is, by swapping the letters.

Gérard Assayag : If I understand correctly, it is the algebra that evolves itself and which transforms itself, which therefore generates a series

Alain Connes : It creates a passage of time. So that had already been tipped since Hamilton had written utterly prophetic sentences, precisely, and where he was talking about the relationship between algebra and time.

So what struck me earlier was that you were explaining yourself that, precisely, in the work of an interpreter, there is always this present. And then there is the past and the future, etc. So we can see, I will say it roughly speaking, it is a deep, microscopic analysis of time. It's an understanding of time which goes further and further into finesse. But what is quite amazing, is that at the algebraic level, there are exactly the same thing that happens and does it rightly, not only, well sure, an algebraic calculation is done in a linear way, with ordered terms in time, that's nothing.

But what's amazing is the reverse. It's the fact that even if we were doing math outside of time, etc., well time would be there and would be present. It would be generated naturally.

Gérard Assayag : You mentioned another point earlier which was without saying it, I will say the technical term, you will excuse me, the Curry-Howard correspondance, i.e. the fact that a proof, we can also look at it as a program, as a calculation.

Alain Connes : Yes, if you want, yes, of course.

Gérard Assayag : It brings up a question we asked ourselves here during the very first Mathematics and Music congress which was organized in 1999 at the request of the European Mathematical Society with M. Bourguignon. We decided to put this under the umbrella of the question "Is there a correspondence between what musicians call musical logic, which is always an organizational logic, and what the math just call logic, mathematical logic or formal logic, mathematical logic ?

And we obviously had not decided this question, we had just managed to say the following : there is a lot of logic in the organization of music. There are many formal terms that we generate. There are even things that look like axioms, that is to say starting hypotheses that we give ourselves to generate a material. But there are two things that are not present ; in music, there is no notion of truth : we do not seek that these terms we aggregate, which will eventually form a partition, establish a certain value of truth, that is not the problem. It's not the problem of logic. The problem of musical logic is not the problem of mathematical logic. Do you agree with me ?

Pierre Boulez : I certainly do not agree with that. I said it discreetly but I think so.

Gérard Assayag : We can say it and I think it's easy to establish. Here there is no truth value, so already, it removes a whole computational aspect because often that's what we're looking for. And then there is another problem much deeper, which is as follows : in pure logic, when we unroll a demonstration, I can use a term A for my demonstration. And I have every right to reuse it afterwards, but nothing happens. It doesn't cost me anything. I use it, I can use it a thousand times if I want, if I longed for. When you consider a musical sequence, an element of language musical as a little bit like a demonstration and that we look at the terms that we aggregate, notes, chords, etc., well, the fact to have exhibited a musical object is not at all innocent. And the second time that we expose it, it doesn't have the same value at all as the first time we had exposed him. So already, already, we are no longer in this hypothesis. *(Laughs.)* I see, I think I see you coming.

Alain Connes : No, no, in fact, if you want, that means that you don't know a certain part of mathematical development, which is what is called linear logic. In logic, in linear logic, especially listen to Jean-Yves Girard from Marseille, when we used it once, we can no longer use.

So I mean, don't believe that the mathematicians are missing of imagination. They used this logic. It has already appeared to them. But in fact, if you will, well, just bounce a little bit on what you say about what happens in music at the logic level, what I would say, it is that there is indeed for the mathematician a role of the aesthetics, when he watches a demonstration. That is to say a mathematician is able to tell by watching a demonstration how likely it is to be true. He is able, by looking at a formula even obtained by a computer, to tell the chances that it has to be true. So there is a role of aesthetics. But if you want for me, you shouldn't believe at all that quality, well, is a quality necessary for a mathematical statement to be true, to be correct, a demonstration of being correct. But the concept, which is much more interesting and much more difficult to obtain and which is much closer to music, is the notion of meaning, that is, if you want a mathematical statement, you could brick a computer that would make you 36 mathematical statements at the shovel and that would all be correct because he would have made them by making correct demonstrations. It would be easy. However, if you looked all of these statements, most of them would be completely uninteresting because they wouldn't make sense.

What does meaning mean ? The notion of sense is something that is ... which does not respond to logic, because the statement in question is correct. But there is for the mathematician a notion of a statement which is wonderful, which has a meaning. And I think that here, we have a connection with music. Because you told me a musical piece doesn't have to be correct, of course, but it must have meaning. If it doesn't make sense then, well, well, I'll say, we could do anything. We could invent 36 music skins. And there, I think that we touch on an essential point because the notion of correct is a necessary condition. It is a necessary condition for the mathematician, of course. But a mathematician could spend his life doing what Arnold said about odd number axioms or things like that. And that means he would have wasted his time. he would have wasted his time because he would not have found the truth that makes sense. He would not have revealed a part of this mathematical reality, but precisely, things that make sense. And this is an extremely difficult thing to define in mathematics. And I think it's also difficult to define that in music, in a certain way.

Pierre Boulez : Yes, it is very difficult because during history, we see people who have less, especially in the XVIII$^{\text{th}}$ century, vocabulary and in one case, the work is very beautiful and in the other case, the work is going to be completely uninteresting. That is, the same grammar can serve not for the purposes at least, but can serve very different purposes.

Alain Connes : Yes, so, I mean, it just means that when we stick to the level of structure, logic, etc., we don't touch the essential problem and the essential problem for mathematics, so far, really, of course, there is the problem of truth, there is the problem that we can talk long, wide and cross. But there is a problem much more difficult, much more important, which is to see precisely in what sense what we found reveals a little corner of mathematical reality. And that, that means to make sense. Exactly.

Gérard Assayag : The problem you raise is the problem that meet automatic theorem provers, programs that demonstrate theorems, they can demonstrate correct theorems, but they don't know how to say that a theorem is interesting. And so, they can demonstrate billions of internal things without sens. So it's interesting because it can join a problem that we know here, which is computer assisted composition, where we have computer programs that composers use to calculate interesting materials or structures.

But they could calculate billions that would not be of interest. It is ultimately the composer who decides. So could you help us ? How could you, composer, help us to converge in a finer, more interesting way, towards results that are not only correct from the point of view of calculation, but likely to interest the musician ?

Pierre Boulez : The first thing I can answer is a very silly answer, it's because I like it, simply because what you give me, what you offer me, I like it.

Gérard Assayag : This is how we work.

Pierre Boulez : Yes, but the whole reasoning of music is based on that, of course, we're not going to say that stupidly : what pleases to me, so I choose it. You may have a terrible taste, the kitsch, and so to say, I like it too, of course. But what's interesting is that when you have so many possibilities, you can't listen, if you have a thousand possibilities, after a hundred you will be tired or you will have absolutely no judgment. That's what is dangerous in music. The more you listen to the different solutions, the less you have reactions, let's say, to choose things. And so, at some point, two things are needed.

First, narrow the scope of the choice and second, decide : "yes, that, why do I choose it ? Because it looks better to me, for this reason, and this reason". But deep down, you're trying to justify yourself. But the main thing is only ... it's not only, but it's mainly intuition and intuition, well, it exists and it's a gift that you have, even if you are very gifted, you have it one day, you don't have it the next day.

That is to say it is very variable and sometimes, you are very sharp, others times less sharp, because you are more seduced by the... And there are also a difficult question in music that is how to join the abstract structure if we can say, and the concrete object, because the concrete object which is very interesting, is maybe in a completely inept structure. And on the contrary, a very intelligent structure can have objects which are completely uninteresting facts. And so, it's this combination that is not easy either more to organize, which makes the work acquire great validity. But that, that has always been the case. I mean, if you look at the history of music, you have for example two very distinct personalities like Berlioz and Schumann, I take these two examples on purpose. In Berlioz's work, there is a sense of instrumentation which is absolutely remarkable even when he was very young. But the sense of harmony, that is to say of harmonic language, was very primitive per se. So we explain it's because he played the guitar when he was young and therefore the guitar simplified his vocabulary. It wouldn't enchant guitarists, if we say that. But while Schumann on the contrary had a very... much more refined harmonic language.

But his instrumental language was really, let's say, without a lot of meaning, without many colors, even, quite simply.

And so, it's very rare to have people in the same musicians who are also gifted, for the different components. So when you have someone like Wagner, obviously you have it, you have everything.

But Wagner who, let's say, never talked about a system, he always talked about new music, music of the future, etc. But he never codified his language. Not at all even. But he took the language as he found it, and under the influence, in particular of Liszt, he diverted the language of the function on which this language lived, and therefore ultimately, he invented this language very ambiguous where all relationships are possible. In more classic language, let's even say Beethoven, not to mention Mozart, you have chords that made revolving chords, so to speak that helped modulation, helping to go a little bit to a neighboring country, but in Wagner, you are... sometimes you have no idea where you are because he uses only ambiguous things.

This ambiguity became generalized gradually and led to Schönberg, who has again created a dogma.

And this dogma was interesting in a certain way, because it did indeed he organized musical language in another way. But this dogma, this dogma, ignored vertical phenomena, and, or barely took into account vertical phenomena, and this is the weakness of the twelve-tone language of Schönberg. Is that one dimension prevails over the others or over the other, specifically, that is, the horizontal domain prevails over the domain vertical and in Bach, that was typical, the vertical and horizontal domains rates will be completely controlled.

And there, the vertical domain, you perceive it immediately ; the horizontal domain, counterpoint, you perceive it when you have studied the score is the difference. You do not perceive the music of the same way if it is written in one way or another. And that, there is nothing to do, we will never change that, because it is a phenomenon of perception.

Alain Connes : Yes, what I wanted to say is at the general level of structure. It's, well, finally, if you want, we can roughly summarize a little bit of the mathematician's work saying that from time to time there is a mathematician who ... finds a big phenomenon. An example of that is for instance when Riemann finds the relation between prime numbers and zeros of a certain function. Okay ? And it's a find, that is to say that it is something that afterwards, we will be able to verify until a certain level with a computer, etc.

But it will give mathematicians a century later, two centuries after a kind of objective. And the reason is that we know that this phenomenon is deep enough and mysterious enough to be sure that all the concepts that will be invented, discovered during this research, that is, to try to find a demonstration of this fact, will have meaning, will have a lot of meaning. So what ? Precisely, where I think that there is a connection that is possible, if you want with music, is that we can say in fact that there are two aspects in the work of the mathematician. That is to say, of course, there is an incredibly rational aspect which consists, once we have an idea of a demonstration, to try to verify that it is correct, of course. That is pure rationalism. But there is an aspect which is much more interesting and which has to do with intuition. And this aspect that has to do with intuition is that there is a period in which the mathematician must absolutely not say to himself "Is what I say correct ? etc. Have I checked all the little details ? etc." And in which, precisely, he must allow himself to dream ? He must afford to see much further and in that period, which is basically, set a little bit like a poetic impulse. It is something that is not transferable in words. That is to say that if a mathematician is in this period, he is unable to explain it to people he is going to meet who will say "yes, well, but then ?".

And he is unable to write it. Because if he writes it, it's like he's said to catch something that will disappear from the moment it goes write it down. But the question I ask myself is to what extent, precisely, this intuition which is terribly present, which is something extremely strong, can be translated in another way. Can it express itself in a musical form, can it express itself otherwise. Because it comes from something that is very deep, which is inside. And if you want, there is a text by Grothendieck that I will read to you if I have time and who speaks precisely of the dream in mathematics and who says to what point, precisely, the dream is not admitted in mathematics. It is not admitted. Why ? Because when a mathematician writes an article, he will not write about dreams he has fantasized, etc. He will write demonstrations. And so there is an invisible part of the mathematician's work which is never visible.

The visible part is going to be a rigorous, written demonstration, etc. And it is going to be a whole... something that is completely hidden and that is all this invisible part and that consisted of these... all these days, etc. in which there was a dream, which was present in intuition, which was present in mind and not yet realized. Well, this, it makes me think if you want the music to work, it feels as if we are at this level of intuition, of something that is not yet realized, etc. but that we managed to transmit, on the other hand. We managed to transmit it in musical form and from the moment when, precisely, there had been something real behind it, there was a real inspiration, etc., there, that makes sense and finally, you get through music to convey someone. So what ? The funny thing is that it finally happened to me to have an outside contribution through a musical work for a problem that I was asking myself and that this musical contribution is more important than if I had read a mathematical text.

I used to listen to relatively short musical works, but that had a meaning, and it was a meaning that fit in with some kind of intuition that I had at one time, but could not translate otherwise, I couldn't translate it into words. I couldn't say "Good, well, etc.". But on the other hand, there was for example, I don't know, a Prelude, which corresponded exactly to this intuition. I did not know why. So there, there is something, in my opinion, if you will, in the notion of meaning and all that.

Pierre Boulez : No, I say that the transcription of a musical intuition, from mathematics to music, is very, very uncomfortable. It is very, very uncomfortable because the choices are not the same. The culture is not the same and the choices are not the same. I was saying just now, I take the case of a composer who did it, Xenakis for not to name it, which used a lot of glissandos, curves, so we saw superb, magnificent curves, etc. But what do we hear, we hear an extremely poor material.

Alain Connes : That was not what I was talking about at all. If you want, there are two very, very different things. There is the fact of using mathematics, well, I remember listening, in fact, to a conference of Xenakis, a very, very long time ago, at a given moment when I was asking to myself if I was going to do math or if I was going to be interested in music ? Things like that.

And he disgusted me, really, because he had come to the Sorbonne, he had made a presentation and in his presentation, he had surrounded the painting in which he had some general formulas by mathematical formulas, and these mathematical formulas had nothing to do with what he was talking about. So, they were there only as a psychological tool for, how to say, scaring people who didn't know math and for, so, imposing something on them like that. So it was not that at all I was talking about.

What I was talking about was a problem that is completely open to my opinion, which is that there are certain mathematical notions, certain mathematical intuitions which are not transmissible by words at the moment.

Pierre Boulez : Yes, but what I wanted to say is not just as a criticism. But let's say a glissando, which follows a curve or another, it's an extremely primitive material, it's a limoth. What interests us in a continuity like that is the notion of cutoff, i.e. the interval, because the interval really defines the way you perceive things. And so when we target, for example, we had seen, even a curve that inspires you a kind of gesture... But what gesture, it should be transmitted not by a direct gesture like that, but you have to transmute it, practically, with intervals that will really give it sense. And that's why I say it's the transposition, or trans-figuration of that, and it's really less primitive than we think.

Alain Connes : Okay, but what I had in mind, for example, you mentioned, about Wagner, the ambiguity between the tones, etc. And then, precisely, there is a mathematical idea which is relatively simple to explain, which is due to Galois and which is not yet, how to say, captured mathematically. And this is precisely the idea of ambiguity. And so, what I have in mind, this is the next thing, is that precisely, like math can capture concepts at levels of conceptualization which are very high... For example, what Galois did, what he understood, is that in fact, people before him, were looking for symmetries, and he, he managed to understand that in fact the first thing to do was to break full symmetry between the roots. And after, once we broke completely symmetry, we managed to find the interior structure by other processes. But what I have in mind is that this idea, well, you go and read 36 math texts around this idea. There is none of these texts which completely exhaust him. There are none. That is to say during that you write it in rational terms, etc., you can't exhaust it. And I am persuaded that there are surely certain musical structures which would arrive at transmit part of the content of this idea, in a complementary way, in the rational way of saying it. That's what I have in mind, not at all that we can use mathematics to guide certain things... It is something which is much more, which is much more at a conceptual level, and to the fact, precisely, that there are mathematical concepts much more elaborate, much more complicated and much more ticks, how to say ? And at the same time much more childish than one could believe and that, precisely, we cannot perceive them completely when we only use linear, rational language, etc. And that polyphonic music etc. can help considerably, if only by the polyphony, that is to say the written language is a unique linear language. There is only one, only one narrator. And polyphony, precisely, well, well, we know that. And in my opinion, precisely, that should allow us to go tobeyond certain things that we are only able to do at the moment.

Gérard Assayag : The question you are asking is really the one concerning the source of creativity. In other words, if I transform it a little, "Are there very deep, pre-verbal levels of representation, almost conceptual, but we're not really going to say that since they are still not verbalized, but which could then, by the time they hatch and where they appear, transform in various ways into mathematics, into language."

Alain Connes : It's exactly that. But what I mean, is that I always come back to Grothendieck, but he shows well how precisely, the process of creativity is a process of back to childhood. It is in this sense that it's a process which is to try to get rid of all dogmas, everything what was imposed on us, etc. And to return to a perception completely childish. But precisely, well, after precisely, having been able to make it universal and to transmit it. So that's obviously at the heart of music, but it's also, it's similar within mathematics.

Pierre Boulez : But is it possible to be so childish ? I was going to say infantile, excuse me, for being so childish, having done all the same experiences that have marked you ?

Alain Connes : Exactly ...

Pierre Boulez : Isn't it artificial ?

Alain Connes : I don't think it's artificial. I do not think this let it be artificial : the example of Grothendieck, which is an extreme example, mentally striking because at one point, precisely, it has, to return to the CNRS because he had left and he had made a request to the CNRS and his text was called *Children's drawings*. So you read this, it's a child, you can say, it's infantile, etc. But in fact, it was connected to one of the deepest math problems which is what's called understanding of the Galois group of the algebraic closure of Q, etc. And it is very often the case, in fact, that when people become professionals, they surround themselves more and more with a protective layer which precisely prevents them from returning to this state. And on the contrary, I think that what is absolutely essential, precisely, is to allow the dream, to allow, to try to go beyond the prohibition of the dream, etc., and to return to that source. And I think when we go back to the source, for example, of the notion of ambiguity, which is a notion that exists and that could be manifested in quite a few areas, well then it will have effectively various forms, it will take various forms. And we will not arrive never to sum it up to an expression.

There will never be a single expression that will sum it up and it will remain a constant source of inspiration. And this is the case for Galois theory, that is to say that it is a theory which is not exhausted and it is not exhausted at the sense where it stays... that's when people really understand it, that is to say that someone could read a book on Galois theory and understand nothing about it just because he would not have understood the initial idea. And it's an idea, precisely, which is a childish idea, which is the idea of ambiguity.

But this idea, when understood, sets things in motion.

It's a real idea, it puts things in motion, and I think, it's very similar to music. Because you get the impression, if you want my impression, me, on creativity in music, it is not an impression, it's more than an impression compared to classical music, romantic music, that is music that is emotional. But my impression was more, compared to the mathematician, that there was a kind of emotional battery that charges, regardless of the instrumental expression and then, once it's charged enough, there is a job which is extremely difficult, which is to make individual emotion universal, transform it, and to make it universal. And it's a process that can seem extremely different from the mathematical process. But schematizing it, it is the same because what does the mathematician do ? What is the role of mathematician's intuition ? The role of his intuition, he is exactly like a hunter. He says "There is something there !". He feels it very, very deeply. But after that thing, he has to go to get it and there is a a reality which is extremely cruel, etc., and which prevents him from going to seek it.

So afterwards, he has a real job, and this job, I think it's the same. It's very similar to the work of having a personal emotion, trying to make it universal. So there is a parallel, of course, these are different things but the role of intuition is the absolutely driving role at startup and it's the same in both, I think.

Pierre Boulez : Yes, I also think so, most certainly, but, in more than that, I would say that there are two constraints : first, the object does not exist, whatever you imagine, so it remains to be built, and secondly, what we have, in music that is instrumental, for example, we have to take into account what is transmission. And this transmission will hurt if for example, the idea is brilliant but the realization is insufficient. And until this difference between the objects you use, for example, the notes. When you have, for example, a very remarkable object, I think, quite simply, because everyone knows that, to the sound of a tom-tom. A tom-tom sound is much more interesting than a violon sound, just like that, but what does it do ? This sound is so interesting that it gets out of context automatically, so you have to restrict it on the contrary, to use it in a very very measured way so that it has its place.

While you have a F#, a G, or whatever, it is neutral, and therefore you can use it for your discovery, that is to say that there are objects that are ready for discovery, and objects that are not ready for discovery, which monopolize ...

Alain Connes : It's a bit like a Chinese character that makes sense in itself as opposed to a letter of the alphabet that has no meaning in itself.

Pierre Boulez : And it is not convenient to have to use both.

Gérard Assayag : We could continue this amazing discussion for a very long time but we have to make the antenna, so that the Festival and the Symposium continue. I think we had two very nice ending words and I would just like to mention a conclusion about emotion, I remember reading in one of your works, the one with Changeux, and which is that so that one day the machines can imagine goals, and therefore become more interesting, they should suffer. We have a great program as computer scientists, to make sure that machines can suffer, too. Thank you Alain Connes, thank you Pierre

Boulez.

Meeting between two major figures of musical creation and contemporary mathematics, Pierre Boulez and Alain Connes.

What is the place of intuition in mathematical reasoning and in artistic activity ? Is there an aesthetic dimension in mathematical activity ? The concept elegance of a mathematical demonstration or a theoretical construction in music does it play a role in creativity ?

This dialogue around invention in the two disciplines is led by Gérard Assayag, director of the CNRS / Ircam Sciences and technologies of music and sound laboratory.

Introduction : Frank Madlener, director of IRCAM.

As part of the third international conference Mathematics and Computation in Music (MCM 2011), Agora 2011.

Capture and postproduction Year Zero. Ircam production.