

Petit mémo des expériences avec le programme en langage Rust

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J'ai voulu tester le programme dont Alain Connes avait parlé dans une des 3 conférences de juillet 2026, un programme écrit par un dénommé Ronnie Andrews dans le langage Rust, un langage plutôt compliqué d'utilisation mais qui permet de faire des calculs en très haute précision. Je voulais uniquement voir les parties imaginaires des zéros non triviaux de la fonction ζ de Riemann calculés assez précisément par programme. L'ia gemini m'a aidée à mettre au point l'installation de Rust, la compilation, certains raffinements, etc.

Ce qui est marrant, c'est qu'en fait, plus la valeur de λ est petite, et mieux le programme se comporte. Voici ci-dessous les résultats des exécutions dans lesquelles je n'ai fait varier que λ de 10 à 2 par valeurs décroissantes. On reproduit la totalité des messages en sortie d'exécution pour la première exécution, ensuite, on se contente du tableau de valeurs. Il s'agit de bien observer les valeurs en regard. J'ai noté le paramètre modifié en rouge dans l'instruction de lancement du programme.

Avec $\lambda^2 = 100$

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/c/Users/Denise_Vella/.cargo/bin/cargo run -release -features hp -p ccm-reproduction - run -lambda-sq 100 -n-modes 150 -precision-digits 100 -display-digits 10 -top 10
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Finished 'release' profile [optimized] target(s) in 0.18s
Running 'target\x86_64-pc-windows-gnu\release\ccm-reproduction.exe run —lambda-sq 100 —n-modes 150
—precision-digits 100 —display-digits 10 —top 10'
runtime parallelism: Rayon workers=16, available CPUs=16, RAYON_NUM_THREADS=<unset>
CCM operator: lambda^2=100, N=150, matrix_size=301
prime powers k <= 100: 35 entries
precision: 100 decimal digits
root acquisition: independent CCM discovery (indices 1..=10)
research capture: claim (requested roots and native artifacts)
[HP] Precomputing 17 unique GL node tables (npts up to 648, prec=397 bits)...
[HP] GL tables ready: 17 total (17 reused from workstation)
[HP] Computing alpha_L, beta_L, gamma_L integrals...
cache artifact: ccm_archimedean_integrals (computed, source=workstation)
cache artifact: ccm_prime_component (computed, source=workstation)
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  cache artifact: ccm_tau_matrix (computed, source=workstation)
[HP] phase timing: tau construction/reuse=2.822s
  cache artifact: ccm_even_sector_matrix (computed, source=workstation)
  cache artifact: ccm_factorization (computed, source=workstation)
[HP] even LU factorization: computed in 0.338s
  cache artifact: ccm_weil_eigenpair (computed, source=workstation)
[HP] phase timing: Weil eigenstate construction/reuse=43.118s
[HP] eigenstate provenance: unshifted limit reached (2000/2000 steps), shifted refinement=RejectedEi
genvalueJump, final relative Tau residual=6.583531351e-120
  cache artifact: ccm_secular_source (computed, source=workstation)
  cache artifact: ccm_root_discovery_window (computed, source=workstation)
[HP] root status summary: 10 total; 10 converged, 0 stagnated, 0 approximate, 0 failed
[HP] phase timing: root discovery/refinement=0.397s
  cache artifact: ccm_convergence_diagnostics (computed, source=workstation)
built and solved in 46.371s, smallest Weil eigenvalue epsilon_N = -9.33663e-120

```

k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status
1	1.045009660	14.13472514	1.3090e1	3.3357e-2	converged
2	12.62957281	21.02203964	8.3925e0	3.9879e-1	converged
3	14.13472514	25.01085758	1.0876e1	3.6165e-1	converged
4	17.43213810	30.42487613	1.2993e1	3.6953e-1	converged
5	18.75308811	32.93506159	1.4182e1	3.6592e-1	converged
6	21.02203964	37.58617816	1.6564e1	3.5586e-1	converged
7	25.01085758	40.91871901	1.5908e1	4.1031e-1	converged
8	28.19075399	43.32707328	1.5136e1	4.5674e-1	converged
9	30.42487613	48.00515088	1.7580e1	4.3626e-1	converged
10	32.93506159	49.77383248	1.6839e1	4.7069e-1	converged

total claim **and** research capture time: 46.382s

Avec $\lambda^2 = 50$

```

/c/Users/Denise_Vella/.cargo/bin/cargo run -release -features hp -p ccm-reproduction - run -lambda-sq 50 -n-modes 150
-precision-digits 100 -display-digits 10 -top 10

```

```

Finished 'release' profile [optimized] target(s) in 0.23s
Running 'target\x86_64-pc-windows-gnu\release\ccm-reproduction.exe run --lambda-sq 50 --n-modes 150
--precision-digits 100 --display-digits 10 --top 10'
runtime parallelism: Rayon workers=16, available CPUs=16, RAYON_NUM_THREADS=<unset>
CCM operator: lambda2=50, N=150, matrix_size=301
prime powers k <= 50: 23 entries
precision: 100 decimal digits
root acquisition: independent CCM discovery (indices 1..=10)
research capture: claim (requested roots and native artifacts)
[HP] Precomputing 17 unique GL node tables (npts up to 648, prec=397 bits)...
[HP] GL tables ready: 17 total (17 reused from workstation)
[HP] Computing alpha_L, beta_L, gamma_L integrals...
cache artifact: ccm_archimedean_integrals (computed, source=workstation)
cache artifact: ccm_prime_component (computed, source=workstation)
cache artifact: ccm_tau_matrix (computed, source=workstation)
[HP] phase timing: tau construction/reuse=2.780s
cache artifact: ccm_even_sector_matrix (computed, source=workstation)
cache artifact: ccm_factorization (computed, source=workstation)
[HP] even LU factorization: computed in 0.409s
cache artifact: ccm_weil_eigenpair (computed, source=workstation)
[HP] phase timing: Weil eigenstate construction/reuse=46.638s
[HP] eigenstate provenance: unshifted limit reached (2000/2000 steps), shifted refinement=RejectedEigenvalueJump,
final relative Tau residual=3.235119971e-120
cache artifact: ccm_secular_source (computed, source=workstation)
cache artifact: ccm_root_discovery_window (computed, source=workstation)
[HP] root status summary: 10 total; 10 converged, 0 stagnated, 0 approximate, 0 failed
[HP] phase timing: root discovery/refinement=0.462s
cache artifact: ccm_convergence_diagnostics (computed, source=workstation)
built and solved in 49.912s, smallest Weil eigenvalue epsilon_N = 7.42561e-120

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k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status
1	3.193333716	14.13472514	1.0941e1	1.1121e-1	converged
2	8.316409980	21.02203964	1.2706e1	2.1868e-1	converged
3	14.13472514	25.01085758	1.0876e1	3.6165e-1	converged
4	16.51852973	30.42487613	1.3906e1	3.4002e-1	converged
5	21.02203964	32.93506159	1.1913e1	4.4164e-1	converged

6	21.68109403	37.58617816	1.5905e1	3.7349e−1	converged
7	25.01085758	40.91871901	1.5908e1	4.1031e−1	converged
8	27.67237247	43.32707328	1.5655e1	4.4211e−1	converged
9	30.42487613	48.00515088	1.7580e1	4.3626e−1	converged
10	32.93506159	49.77383248	1.6839e1	4.7069e−1	converged
total claim and research capture time: 49.926s					

avec $\lambda^2 = 30$

[HP] phase timing: root discovery/refinement=0.380s					
cache artifact: ccm_convergence_diagnostics (reused, source=workstation)					
built and solved in 0.723s, smallest Weil eigenvalue epsilon_N = 1.19227e−119					
k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status
1	7.711385393	14.13472514	6.4233e0	3.4253e−1	converged
2	10.60976620	21.02203964	1.0412e1	3.0513e−1	converged
3	14.13472514	25.01085758	1.0876e1	3.6165e−1	converged
4	21.02203964	30.42487613	9.4028e0	5.0997e−1	converged
5	25.01085758	32.93506159	7.9242e0	6.1870e−1	converged
6	30.42487613	37.58617816	7.1613e0	7.2004e−1	converged
7	32.93506159	40.91871901	7.9837e0	7.0972e−1	converged
8	37.58617816	43.32707328	5.7409e0	8.7778e−1	converged
9	40.91871901	48.00515088	7.0864e0	8.3086e−1	converged
10	43.32707328	49.77383248	6.4468e0	8.8766e−1	converged
total claim and research capture time: 0.734s					

avec $\lambda^2 = 25$

```
$ /c/Users/Denise_Vella/.cargo/bin/cargo run --release --features hp -p ccm-reproduction -- run --lambda-sq 25
--n-modes 150 --precision-digits 100 --display-digits 10 --top 10
Finished 'release' profile [optimized] target(s) in 0.19s
Running 'target\x86_64-pc-windows-gnu\release\ccm-reproduction.exe run --lambda-sq 25 --n-modes 150
--precision-digits 100 --display-digits 10 --top 10'
runtime parallelism: Rayon workers=16, available CPUs=16, RAYON_NUM_THREADS=<unset>
CCM operator: lambda^2=25, N=150, matrix_size=301
prime powers k <= 25: 14 entries
precision: 100 decimal digits
root acquisition: independent CCM discovery (indices 1..=10)
research capture: claim (requested roots and native artifacts)
[HP] Precomputing 17 unique GL node tables (npts up to 648, prec=397 bits)...
[HP] GL tables ready: 17 total (17 reused from workstation)
[HP] Computing alpha_L, beta_L, gamma_L integrals...
cache artifact: ccm_archimedean_integrals (computed, source=workstation)
cache artifact: ccm_prime_component (computed, source=workstation)
cache artifact: ccm_tau_matrix (computed, source=workstation)
[HP] phase timing: tau construction/reuse=2.761s
cache artifact: ccm_even_sector_matrix (computed, source=workstation)
cache artifact: ccm_factorization (computed, source=workstation)
[HP] even LU factorization: computed in 0.345s
cache artifact: ccm_weil_eigenpair (computed, source=workstation)
[HP] phase timing: Weil eigenstate construction/reuse=44.997s
[HP] eigenstate provenance: unshifted limit reached (2000/2000 steps), shifted refinement=Accepted,
final relative Tau residual=1.179726030e-120
cache artifact: ccm_secular_source (computed, source=workstation)
cache artifact: ccm_root_discovery_window (computed, source=workstation)
[HP] root status summary: 10 total; 10 converged, 0 stagnated, 0 approximate, 0 failed
[HP] phase timing: root discovery/refinement=0.503s
cache artifact: ccm_convergence_diagnostics (computed, source=workstation)
built and solved in 48.293s, smallest Weil eigenvalue epsilon_N = 2.22368e-119
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k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status
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1	14.13472514	14.13472514	3.4143e−117	1.1762e2	converged
2	21.02203964	21.02203964	3.0185e−114	1.1484e2	converged
3	25.01085758	25.01085758	2.3467e−112	1.1303e2	converged
4	30.42487613	30.42487613	1.9767e−109	1.1019e2	converged
5	32.93506159	32.93506159	4.0027e−108	1.0892e2	converged
6	37.58617816	37.58617816	6.9571e−106	1.0673e2	converged
7	40.91871901	40.91871901	7.7989e−104	1.0472e2	converged
8	43.32707328	43.32707328	1.0714e−102	1.0361e2	converged
9	48.00515088	48.00515088	5.2679e−100	1.0096e2	converged
10	49.77383248	49.77383248	5.9148e−99	9.9925e1	converged
total claim and research capture time: 48.307s					

Avec $\lambda^2 = 15$

built and solved in 46.127s, smallest Weil eigenvalue epsilon_N = 6.83428e−70					
k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status
1	14.13472514	14.13472514	5.1726e−66	6.6437e1	converged
2	21.02203964	21.02203964	1.0580e−62	6.3298e1	converged
3	25.01085758	25.01085758	1.0576e−60	6.1374e1	converged
4	30.42487613	30.42487613	1.0565e−57	5.8459e1	converged
5	32.93506159	32.93506159	2.2032e−56	5.7175e1	converged
6	37.58617816	37.58617816	3.7829e−54	5.4997e1	converged
7	40.91871901	40.91871901	4.0144e−52	5.3008e1	converged
8	43.32707328	43.32707328	5.1906e−51	5.1922e1	converged
9	48.00515088	48.00515088	2.1701e−48	4.9345e1	converged
10	49.77383248	49.77383248	2.2607e−47	4.8343e1	converged

Avec $\lambda^2 = 14$

built and solved in 0.754s, smallest Weil eigenvalue epsilon_N = 1.35318e-64						
k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status	
1	14.13472514	14.13472514	9.8746e-61	6.1156e1	converged	
2	21.02203964	21.02203964	1.9262e-57	5.8038e1	converged	
3	25.01085758	25.01085758	1.8572e-55	5.6129e1	converged	
4	30.42487613	30.42487613	1.7489e-52	5.3240e1	converged	
5	32.93506159	32.93506159	3.5344e-51	5.1969e1	converged	
6	37.58617816	37.58617816	5.6863e-49	4.9820e1	converged	
7	40.91871901	40.91871901	5.7278e-47	4.7854e1	converged	
8	43.32707328	43.32707328	7.1114e-46	4.6785e1	converged	
9	48.00515088	48.00515088	2.7284e-43	4.4245e1	converged	
10	49.77383248	49.77383248	2.7446e-42	4.3259e1	converged	
total claim and research capture time: 0.768s						

Avec $\lambda^2 = 13$

built and solved in 0.759s, smallest Weil eigenvalue epsilon_N = 3.18607e−59						
k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status	
1	14.13472514	14.13472514	2.2283e−55	5.5802e1	converged	
2	21.02203964	21.02203964	4.1127e−52	5.2709e1	converged	
3	25.01085758	25.01085758	3.8024e−50	5.0818e1	converged	
4	30.42487613	30.42487613	3.3421e−47	4.7959e1	converged	
5	32.93506159	32.93506159	6.5110e−46	4.6704e1	converged	
6	37.58617816	37.58617816	9.7087e−44	4.4588e1	converged	
7	40.91871901	40.91871901	9.2012e−42	4.2648e1	converged	
8	43.32707328	43.32707328	1.0894e−40	4.1600e1	converged	

9	48.00515088	48.00515088	3.7793e−38	3.9104e1	converged
10	49.77383248	49.77383248	3.6490e−37	3.8135e1	converged
total claim and research capture time: 0.772s					

Avec $\lambda^2 = 12$

built and solved in 0.765s, smallest Weil eigenvalue epsilon_N = 4.71608e−54					
k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status
1	14.13472514	14.13472514	3.1383e−50	5.0654e1	converged
2	21.02203964	21.02203964	5.4284e−47	4.7588e1	converged
3	25.01085758	25.01085758	4.7776e−45	4.5719e1	converged
4	30.42487613	30.42487613	3.8728e−42	4.2895e1	converged
5	32.93506159	32.93506159	7.2265e−41	4.1659e1	converged
6	37.58617816	37.58617816	9.8542e−39	3.9581e1	converged
7	40.91871901	40.91871901	8.6921e−37	3.7673e1	converged
8	43.32707328	43.32707328	9.7308e−36	3.6649e1	converged
9	48.00515088	48.00515088	2.9978e−33	3.4204e1	converged
10	49.77383248	49.77383248	2.7576e−32	3.3256e1	converged
total claim and research capture time: 0.779s					

Avec $\lambda^2 = 5$

built and solved in 0.762s, smallest Weil eigenvalue epsilon_N = 9.52118e−18					
k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status
1	14.13472514	14.13472514	2.5013e−14	1.4752e1	converged
2	21.02203964	21.02203964	1.2603e−11	1.2222e1	converged

3	25.01085758	25.01085758	4.2908e−10	1.0766e1	converged
4	30.42487620	30.42487613	7.1153e−8	8.6310e0	converged
5	32.93506215	32.93506159	5.6176e−7	7.7681e0	converged
6	37.58619055	37.58617816	1.2395e−5	6.4818e0	converged
7	40.91896173	40.91871901	2.4271e−4	5.2268e0	converged
8	43.32788791	43.32707328	8.1463e−4	4.7258e0	converged
9	48.02241436	48.00515088	1.7263e−2	3.4442e0	converged
10	49.82654878	49.77383248	5.2716e−2	2.9751e0	converged
total claim and research capture time: 0.775s					

Avec $\lambda^2 = 4$

k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status
1	14.13472514	14.13472514	1.3127e−9	1.0032e1	converged
2	21.02204001	21.02203964	3.7141e−7	7.7528e0	converged
3	25.01086557	25.01085758	7.9853e−6	6.4958e0	converged
4	30.42546732	30.42487613	5.9119e−4	4.7115e0	converged
5	32.93800478	32.93506159	2.9432e−3	4.0488e0	converged
6	37.60859061	37.58617816	2.2412e−2	3.2245e0	converged
7	41.08949075	40.91871901	1.7077e−1	2.3795e0	converged
8	43.62250730	43.32707328	2.9543e−1	2.1663e0	converged
9	48.64495328	48.00515088	6.3980e−1	1.8752e0	converged
10	52.34436593	49.77383248	2.5705e0	1.2870e0	converged

Avec $\lambda^2 = 3$

k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status
1	14.13477144	14.13472514	4.6300e−5	5.4847e0	converged
2	21.02653193	21.02203964	4.4923e−3	3.6702e0	converged

3	25.04941669	25.01085758	3.8559e−2	2.8120e0	converged
4	30.88751548	30.42487613	4.6264e−1	1.8180e0	converged
5	34.59035950	32.93506159	1.6553e0	1.2988e0	converged
6	39.53082848	37.58617816	1.9447e0	1.2862e0	converged
7	43.77907975	40.91871901	2.8604e0	1.1555e0	converged
8	49.52871933	43.32707328	6.2016e0	8.4425e−1	converged
9	55.00896260	48.00515088	7.0038e0	8.3595e−1	converged
10	60.08251528	49.77383248	1.0309e1	6.8380e−1	converged

Avec $\lambda^2 = 2$

k	computed eigenvalue	Riemann zero t_k	abs error	matching digits	status
1	14.34766847	14.13472514	2.1294e−1	1.8220e0	converged
2	22.67628532	21.02203964	1.6542e0	1.1041e0	converged
3	31.16731629	25.01085758	6.1565e0	6.0880e−1	converged
4	39.77698908	30.42487613	9.3521e0	5.1232e−1	converged
5	48.47757164	32.93506159	1.5543e1	3.2614e−1	converged
6	57.24806089	37.58617816	1.9662e1	2.8140e−1	converged
7	66.07231068	40.91871901	2.5154e1	2.1132e−1	converged
8	74.93808397	43.32707328	3.1611e1	1.3692e−1	converged
9	83.83619485	48.00515088	3.5831e1	1.2703e−1	converged
10	92.75975702	49.77383248	4.2986e1	6.3675e−2	converged