Modeling Goldbach's conjecture in min-plus algebra by calculating the centers of triangular graphs, Denise Vella-Chemla, may 2026

1. Introduction

Goldbach's conjecture, stated in 1742, stipulates that every even integer $n \geq 4$ is the sum of two prime numbers. Despite massive computer verifications, no general proof has yet been accepted by the mathematical community.

In this note, we propose a graph-theoretic approach : we model Goldbach decompositions as the centers of a particular triangular graph T. We show that the existence of at least two centers in T (one trivial center providing no knowledge and another) would imply the validity of the conjecture. However, we are unable to prove this. Nevertheless, the approach is summarized below because it shows the connection that exists between Goldbach's conjecture, the calculation of shortest paths using the Manhattan distance, and min-plus algebra.

2. Definitions and graph construction

2.1 Definitions.

Let n be an integer pair ≥ 6. Sets X_n and Y_n, triangular graph T :

$$X_n = \{p_k \mid 3 \leq p_k \leq n-3,\ p_k \text{ a prime number}\} \cup \{0, n\},$$
$$Y_n = \{n - p_k \mid p_k \in X_n\} \cup \{0, n\}.$$

Example :

For $n = 8$,

$$\mathrm{X}_{16} = \{0, 3, 5, 8\} \qquad \mathrm{Y}_{16} = \{8, 5, 3, 0\}$$

The grid-graph $G = (S, A)$ is defined as :

- its set of vertices S :
$$S = \{(x, y) \mid x \in X_n, y \in Y_n\}.$$
- its set of edges, consisting of horizontal and vertical edges between adjacent vertices (separated by a Manhattan distance of 1).
- the numbering function f, which enumerates the vertices from first to last in the usual reading order, that is, row by row, and within each row, column by column.

The triangular graph T is the subset of G restricted to the vertices (x, y) with $x \geq y$ (it contains the vertices located below the main diagonal).

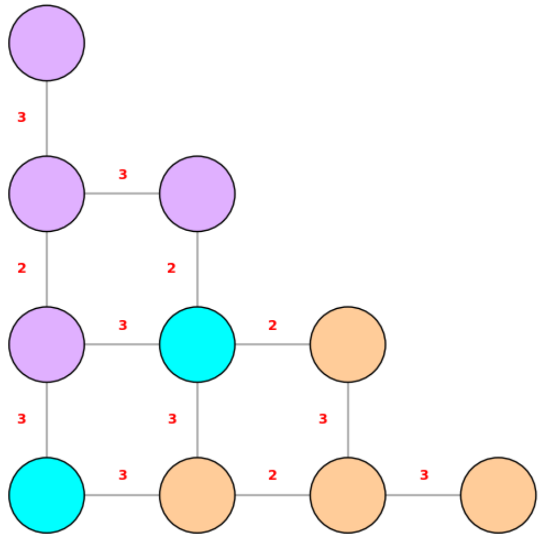


FIGURE 1 : $n = 8$

The vertices are numbered from top to bottom and in each line from left to right, they are in one-to-one correspondence with the points on the Cartesian plane that correspond to them.

2.3 Edges labeling

The edges are oriented from top to bottom and from left to right. They are labeled by the differences between successive prime numbers, which are elements of the ordered sequence $\Delta_n = [s_1, s_2, \ldots, s_k]$, where :

$$\begin{cases} s_1 = 3 \text{ (first gap from 0 to the first odd prime number 3),} \\ s_i = p_{i+1} - p_i \text{ for } 2 \leq i \leq k-1 \text{ (gaps between successive prime numbers),} \\ s_k = n - \max\{p \in X_n \mid 3 \leq p \leq n-3\} \\ \qquad\qquad\qquad \text{(last gap between the greatest prime number } \leq n-3, \text{ and } n). \end{cases}$$

2.4 Distance and eccentricity

Let's define the graph theory concepts involved in the modeling : the concept of *distance between 2 vertices*, the concept of *eccentricity of a vertex*, and the concept of *center of a graph*.

- *Distance between 2 vertices* : in T, the distance between two vertices is the length of the shortest path, a set of successive directed edges connecting these vertices ; the length of a path is the sum of the weights of the edges constituting this path. Since the edges are labeled by the gaps between prime numbers, the length of a path from the source of the graph S (top left) to any point in the graph that is not on the left edge of the triangle is the sum of two prime numbers $p + q$, where the prime number p is equal to the sum of the labels of the vertical edges of the path and the prime number q is equal to the sum of the labels of the horizontal edges of the path.

- *Eccentricity* $e(s)$ *of a vertex* s *: it is defined as follows :*

$$e(s) = \max\{d(s, s') \mid s' \in S_T\}.$$

- *A center of the graph* T : A vertex s is a center of the graph T if

$$e(s) = \min\{e(s') \mid s' \in S_T\}.$$

Example : for $n = 16, \Delta_{16} = [3, 2, 2, 4, 2, 3]$ (that are the gaps between 0 and 3, 3 and 5, 5 and 7, 7 and 11, 11 and 13 and last, 13 and 16).

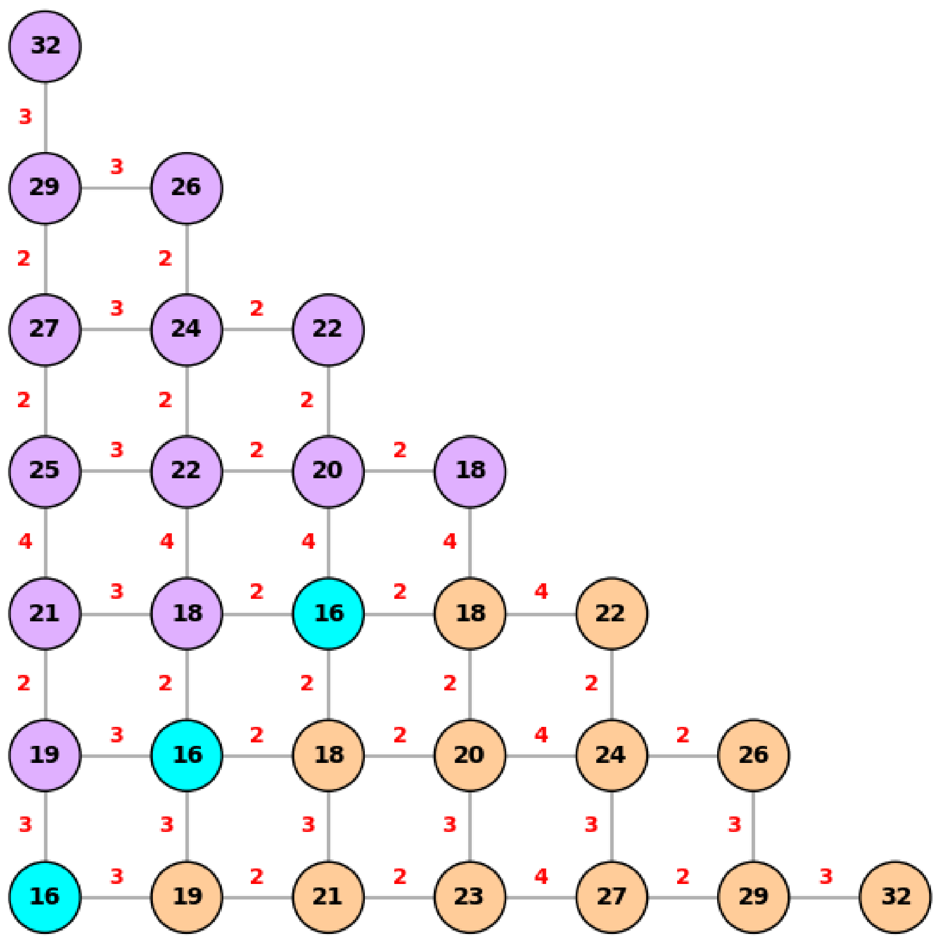


FIGURE 2 : triangular graph T for $n = 16$.

3. Key properties of graph T

3.1 Connexity of T

Lemma 1 : *T is connected.*

Proof :

- T contains the vertices $(0, n)$ (called the "source" S) and $(n, 0)$ (called the "sink" P) ;
- There exists a monotonic path (in which x or y increases when traversing each edge of the path) between any vertex S and (x, y) on the one hand, and between (x, y) and P on the other hand ;

3.2. The eccentricity of any vertex s can be calculated by comparing only the two shortest paths from s to S and from s to P.

Lemma 2 : $\forall s', d(s, s') \leq d(s, P)$ and $d's, s') \leq d(s, S)$.

Proof :

- In a grid graph, the largest distances from s are those from s to the corners S and P.
- The other vertices s' are "closer" to s than S or P, so their distance to s is less than or equal to $\max(d(s, S), d(s, P))$. Three visual proofs of these maximum distances between a point and S or a point and P are provided in the appendix.
- Calculation of eccentricities : we can replace $d(s, P)$ with $2n - p - q$ and $d(s, S)$ with $p + q$ in the calculation of the eccentricity by calculating the maximum above because these values are those we used to define the coordinates of the vertices of the graph, and we can replace the length of a path in the graph with the Manhattan distance separating, in the Euclidean plane, the points corresponding to the two vertices :

$$d((x_1, y_1), (x_2, y_2)) = |x_2 - x_1| + |y_2 - y_1|.$$

3.3. Eccentricity of vertices.

Lemma 3 : *For any vertex $s = (p, q)$ of T, its eccentricity is given by comparing the only 2 shortest paths that lead from s to S and from s to P.*

3.4. Centers and Goldbach decompositions

Theorem 1 : *The vertices of T, with eccentricity n, if any exist, and which would be different from the vertex at the "tip" of the graph T when the graph is embedded in the Euclidean plane (i.e., in the bottom left corner of the diagram), correspond to the Goldbach decompositions of n, that is, the vertices (p, q) with $p + q = n$ and p and q being prime.*

If $\max(2n - p - q, p + q) = n$ then two cases are possible :

- if the maximum is $2n - p - q$, then $2n - p - q = n$ and therefore $n = p + q$;
- if the maximum is $p + q$, then we directly have $n = p + q$.

In conclusion, the centers of the graph T, if their eccentricity (i.e., the minimum of all the eccentricities of the vertices of the graph) is n, correspond to the Goldbach decompositions of n. But we are not certain that a center of eccentricity n exists in the triangular graph without its apex.

4. Using Matrix Calculus in (min, plus) Algebra for Shortest Path Calculation via the Roy-Floyd-Warshall Algorithm

The calculation of shortest paths from any vertex to any vertex in a graph (and therefore, in particular, to the two vertices source S and sink P) is performed using the Roy-Floyd-Warshall algorithm and by performing matrix calculations in the (min, plus) algebra. In this algebra, addition is replaced by calculating the minimum, and multiplication is replaced by addition. The calculation of eccentricities is performed using an adjacency matrix between the vertices of the triangular graph. The presentation of the shortest path calculation algorithm can be found in Bernard Roy's article in the Comptes-Rendus de l'Académie des Sciences in 1959, "Transitivity and Connectivity." (Roy, B., "Transitivity and Connectivity." C. R. Acad. Sci., Paris, vol. 249, 1959, pp. 216-218. See https://gallica.bnf.fr/ark:/12148/bpt6k3201c/f222.item.)

Below is provided the initial matrix of distances between vertices for $n = 10$. All elements of the matrix are initialized to the value $+\infty$ (denoted ∞) because we are looking for the shortest possible path and $\min(x, +\infty) = x$. The elements of the diagonal of the matrix are initialized to 0 (because the distance from any vertex to itself is zero) and the gaps between successive prime numbers (either 3 or the gap between the largest prime number less than or equal to $n-3$ and the number n) are positioned at the appropriate elements of the matrix, due to the numbering of the vertices. Note : the matrix is always of size $\dfrac{\pi(n-3)(\pi(n-3)+1)}{2}$ with $\pi(x)$ the usual notation for the number of prime numbers less than or equal to x.

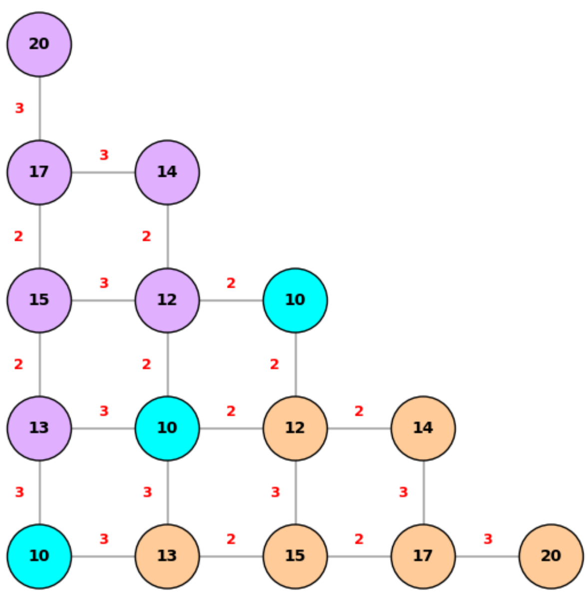


FIGURE 3 : $n = 10$

Below is the vertex numbering and edge labeling.

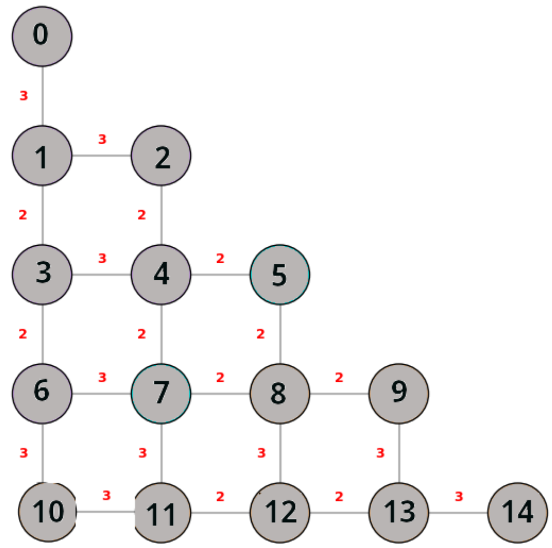


FIGURE 4 : Vertex numbering from 0 to 14

The Roy-Floyd-Warshall algorithm works on a matrix and, as it executes, converts the different elements of the matrix into min-plus algebra using the relation [1] :

$$M_{ij}^{k} = \min(M_{ij}^{k-1}, M_{ik}^{k-1} + M_{kj}^{k-1})$$

Initial matrix for $n = 10$: The adjacency matrix M containing the labels of the edges connecting the vertices is of size 15×15, it is a very sparse and symmetric matrix..

$$M = \begin{pmatrix}
0 & 3 & & & & & & & & & & & & & \\
3 & 0 & 3 & 2 & & & & & & & & & & & \\
 & 3 & 0 & & 2 & & & & & & & & & & \\
 & 2 & & 0 & 3 & & 2 & & & & & & & & \\
 & & 2 & 3 & 0 & 2 & & 2 & & & & & & & \\
 & & & & 2 & 0 & & & 2 & & & & & & \\
 & & & 2 & & & 0 & 3 & & & 3 & & & & \\
 & & & & 2 & & 3 & 0 & 2 & & & 3 & & & \\
 & & & & & 2 & & 2 & 0 & 2 & & & 3 & & \\
 & & & & & & & & 2 & 0 & & & & 3 & \\
 & & & & & & 3 & & & & 0 & 3 & & & \\
 & & & & & & & 3 & & & 3 & 0 & 2 & & \\
 & & & & & & & & 3 & & & 2 & 0 & 2 & \\
 & & & & & & & & & 3 & & & 2 & 0 & 3 \\
 & & & & & & & & & & & & & 3 & 0
\end{pmatrix}$$

The matrix below provides the eccentricities calculated by Dijkstra's algorithm on the initial matrix M.

1. A description of the world of characteristic 1 can be found in section 4.2 of Alain Connes's 2015 article, *An essay on the Riemann hypothesis*, https://arxiv.org/pdf/1509.05576.

Terminal matrix of eccentricities, calculated by the Roy-Floyd-Warshall shortest path algorithm. The Goldbach decompositions $10 = 3 + 7$ and $10 = 5 + 5$ correspond to the two occurrences of the number 10 in the first row.

$$
M' = \begin{pmatrix}
0 & 3 & 6 & 5 & 8 & 10 & 7 & 10 & 12 & 14 & 10 & 13 & 15 & 17 & 20 \\
3 & 0 & 3 & 2 & 5 & 7 & 4 & 7 & 9 & 11 & 7 & 10 & 12 & 14 & 17 \\
6 & 3 & 0 & 5 & 2 & 4 & 7 & 4 & 6 & 8 & 10 & 7 & 9 & 11 & 14 \\
5 & 2 & 5 & 0 & 3 & 5 & 2 & 5 & 7 & 9 & 5 & 8 & 10 & 12 & 15 \\
8 & 5 & 2 & 3 & 0 & 2 & 5 & 2 & 4 & 6 & 8 & 5 & 7 & 9 & 12 \\
10 & 7 & 4 & 5 & 2 & 0 & 7 & 4 & 2 & 4 & 10 & 7 & 5 & 7 & 10 \\
7 & 4 & 7 & 2 & 5 & 7 & 0 & 3 & 5 & 7 & 3 & 6 & 8 & 10 & 13 \\
10 & 7 & 4 & 5 & 2 & 4 & 3 & 0 & 2 & 4 & 6 & 3 & 5 & 7 & 10 \\
12 & 9 & 6 & 7 & 4 & 2 & 5 & 2 & 0 & 2 & 8 & 5 & 3 & 5 & 8 \\
14 & 11 & 8 & 9 & 6 & 4 & 7 & 4 & 2 & 0 & 10 & 7 & 5 & 3 & 6 \\
10 & 7 & 10 & 5 & 8 & 10 & 3 & 6 & 8 & 10 & 0 & 3 & 5 & 7 & 10 \\
13 & 10 & 7 & 8 & 5 & 7 & 6 & 3 & 5 & 7 & 3 & 0 & 2 & 4 & 7 \\
15 & 12 & 9 & 10 & 7 & 5 & 8 & 5 & 3 & 5 & 5 & 2 & 0 & 2 & 5 \\
17 & 14 & 11 & 12 & 9 & 7 & 10 & 7 & 5 & 3 & 7 & 4 & 2 & 0 & 3 \\
20 & 17 & 14 & 15 & 12 & 10 & 13 & 10 & 8 & 6 & 10 & 7 & 5 & 3 & 0
\end{pmatrix}
$$

The first four matrices for the numbers $n = 6, 8, 10, 12$ are provided below, omitting the elements initialized to the value ∞ for ease of reading. The unusual positioning of the elements in the matrices makes it completely incomprehensible what could cause the systematic appearance of the number n after applying the Roy-Floyd-Warshall algorithm (which eventually fills the matrix completely, due to the connected graph). The matrices are symmetric and have determinants and traces of zero.

example 1 : $n = 6$, points $= [0, 3, 6]$, gaps $= [3, 3]$.

$$
M = \begin{pmatrix}
0 & 3 & & & \\
3 & 0 & 3 & & \\
& 3 & 0 & 3 & \\
& & 3 & 0 & 3 \\
& & & 3 & 0
\end{pmatrix}
\qquad
M' = \begin{pmatrix}
0 & 3 & 6 & 9 & 12 \\
3 & 0 & 3 & 6 & 9 \\
6 & 3 & 0 & 3 & 6 \\
9 & 6 & 3 & 0 & 3 \\
12 & 9 & 6 & 3 & 0
\end{pmatrix}
$$

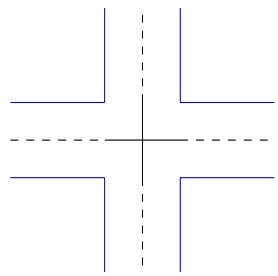
example 2 : $n = 8$, points $= [0, 3, 5, 8]$, gaps $= [3, 2, 3]$.

$$
M = \begin{pmatrix}
0 & 3 & & & & & & & \\
3 & 0 & 3 & 2 & & & & & \\
& 3 & 0 & & 2 & & & & \\
& 2 & & 0 & 3 & & & & \\
& & 2 & 3 & 0 & 2 & 3 & & \\
& & & & 2 & 0 & & 3 & \\
& & & & 3 & & 0 & 2 & \\
& & & & & 3 & 2 & 0 & 3 \\
& & & & & & & 3 & 0
\end{pmatrix}
\qquad
M' = \begin{pmatrix}
0 & 3 & 6 & 5 & 8 & 10 & 11 & 13 & 16 \\
3 & 0 & 3 & 2 & 5 & 7 & 8 & 10 & 13 \\
6 & 3 & 0 & 5 & 2 & 4 & 5 & 7 & 10 \\
5 & 2 & 5 & 0 & 3 & 5 & 6 & 8 & 11 \\
8 & 5 & 2 & 3 & 0 & 2 & 3 & 5 & 8 \\
10 & 7 & 4 & 5 & 2 & 0 & 5 & 3 & 6 \\
11 & 8 & 5 & 6 & 3 & 5 & 0 & 2 & 5 \\
13 & 10 & 7 & 8 & 5 & 3 & 2 & 0 & 3 \\
16 & 13 & 10 & 11 & 8 & 6 & 5 & 3 & 0
\end{pmatrix}
$$

6. Let n tend towards 1

It's important to keep in mind that working within min-plus algebra completely overturns any classical understanding of the concept of distance : below, we show the "unit circle", i.e., the set of points at a min-plus distance of 1 from the origin.

For the min norm, the unit curve (blue) is :



This "unit circle," or rather the unit curve, is neither closed nor connected (one cannot go from any point on it to any other point on it without "leaving" it).

To illustrate the modification of the notion of distance, we see that the point $(0.2, 10)$ (with a distance of 0.2 from the origin) is "closer" to the origin than the point $(0.5, 0.5)$ (with a distance of 0.5 from the origin), even though it is much farther away "as the crow flies".

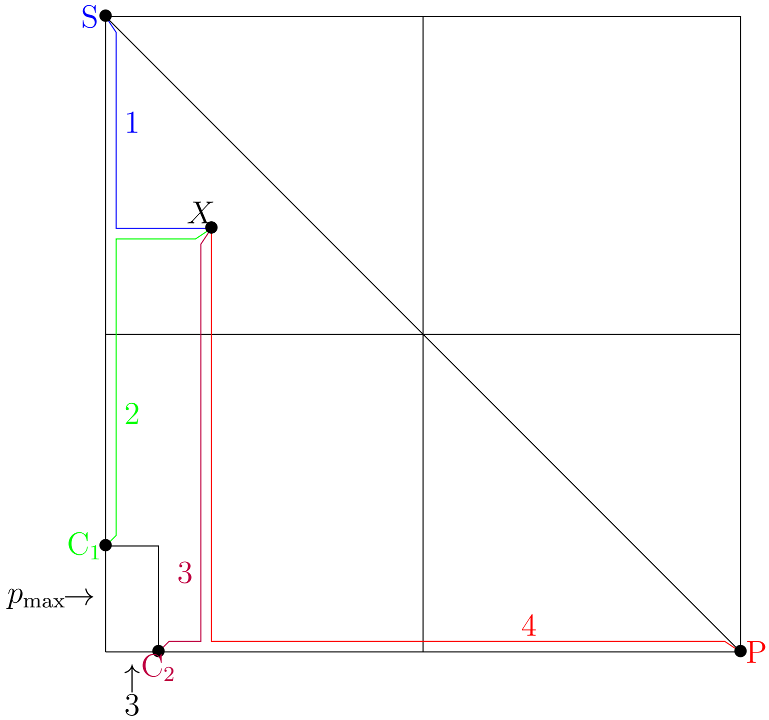
If we scale the Goldbach squares that underlie the triangular (right-angled isosceles) graphs so that their side length is 1, then the points corresponding to the Goldbach decompositions are all at a distance $1/2$ from the points S and P. We can see the correspondence between addition and multiplication as the fact of writing the sum $3 + 5 = 8$ on the hyperbola with equation $xy = 8$ at the point with coordinates $(3, 5)$.

7. Conclusion

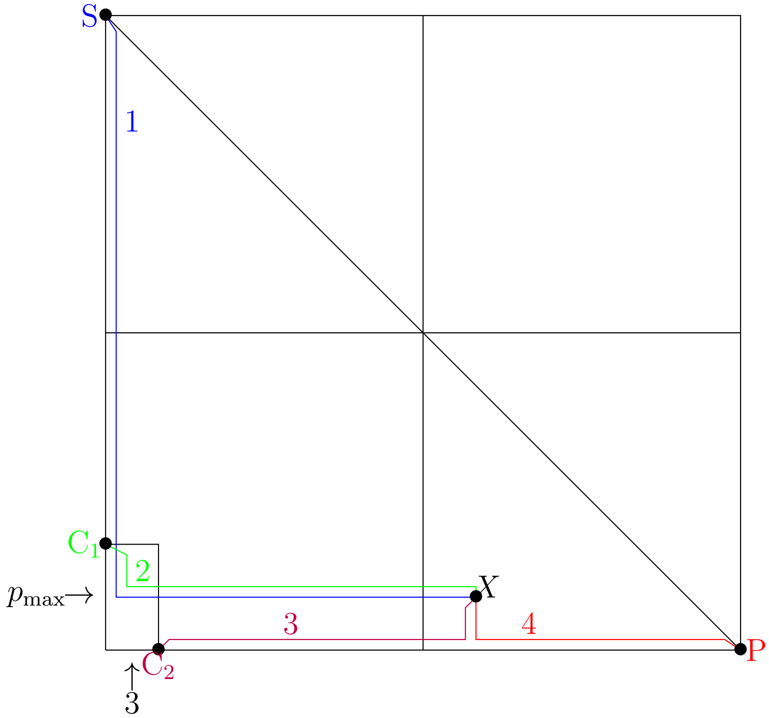
The approach presented here provides a geometric and graphical understanding of certain properties that are difficult to grasp purely algebraically. Unfortunately, the impossibility of proving, without using arithmetic properties of prime numbers such as divisibility, that matrices, once the Roy-Floyd-Warshall algorithm is applied, do indeed contain at least one element equal to n prevents us from using this approach to prove Goldbach's conjecture. We will retain its pedagogical value for introducing Goldbach's conjecture and the notions of vertex, edge, eccentricity, and center in graph theory, as well as the Roy-Floyd-Warshall shortest path algorithm.

Annexe : it suffices to consider distances to source S and to sink P in the triangular graph T

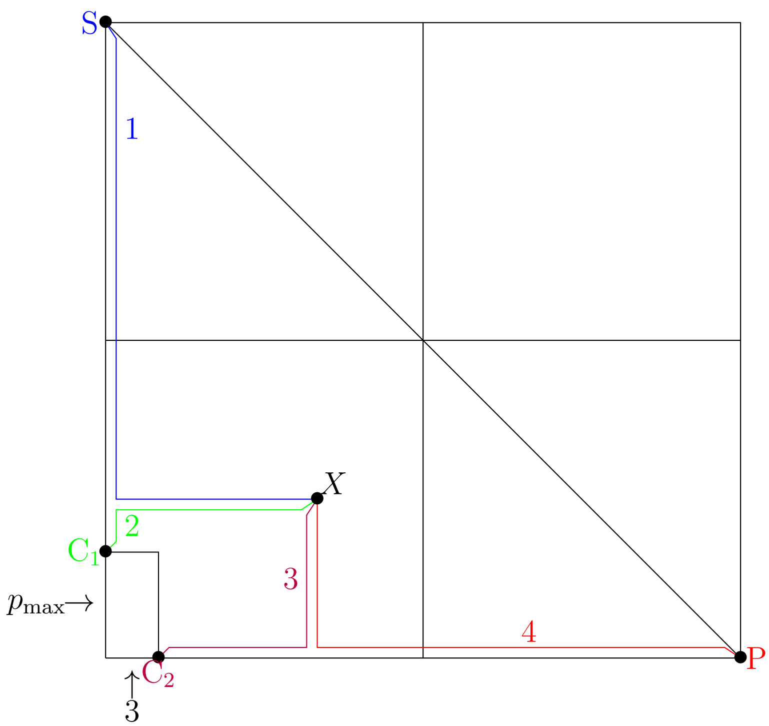
Wordless proofs that we can only consider the lengths of the paths from a point X to S and from this point X to P to calculate its eccentricity (we have drawn as best as possible the paths whose lengths are Manhattan distances in the Euclidean plane, the paths start from the points and end at the points, the small diagonal detours are useful to move the paths away from the edges a little so that they are more visible ; the notation $\ell(k)$ denotes the length of the path k, the numbers of the paths being indicated in color) :



$$\ell(2) \leq \ell(4)$$
$$\ell(3) \leq \ell(4)$$



$\ell(2) \leq \ell(1)$
$\ell(3) \leq \ell(1)$



$\ell(2) \leq \ell(1)$
$\ell(3) \leq \ell(4)$